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Absolute Value Equations & Inequalities

Equations:

Isolate the absolute value expression, then create two equations; one positive, and one negative. Solve each, then check !

1. $|x| = 2$

$x = 2$ $x = -2$

2. $|2x - 9| = 15$

$2x - 9 = 15$ $2x - 9 = -15$

$2x = 24$ $2x = -6$

$x = 12$ $x = -3$

3. $|9 - 2x| = 10 + 3x$

$9 - 2x = 10 + 5x$ $9 - 2x = -10 - 3x$

$9 = 10 + 5x$ $9 + x = -10$

$-1 = 5x$ $x = -19$ extraneous

$-\frac{1}{5} = x$

Check: $|9 - 2(-19)| = 10 + 3(-19)$

$|9 + 38| = 10 - 57$

$|47| = -47$

$47 = -47$ No

Check: $|9 - 2(-\frac{1}{5})| \stackrel{?}{=} 10 + 3(-\frac{1}{5})$

$|9 + \frac{2}{5}| = 10 - \frac{3}{5}$

$|9\frac{2}{5}| = 9\frac{2}{5}$

$9\frac{2}{5}$ ✓

Inequalities:

Create and solve a compound inequality.

If the inequality is < or ≤ then the compound inequality is "and."

$|x - 2| \leq 1$

$x - 2 \leq 1$ AND $x - 2 \geq -1$

$x \leq 3$ AND $x \geq 1$

If the inequality is > or ≥ then the compound inequality is "or."

$|x + 1| \geq 3$ $x + 1 \geq 3$ OR $x + 1 \leq -3$

$x \geq 2$ OR $x \leq -4$

Absolute Value Inequalities

Inequality ($c > 0$)	Equivalent form	Graph of solution
$\lvert ax + b\rvert < c$	$-c < ax + b < c$	
$\lvert ax + b\rvert \leq c$	$-c \leq ax + b \leq c$	
$\lvert ax + b\rvert > c$	$ax + b < -c$ or $ax + b > c$	
$\lvert ax + b\rvert \geq c$	$ax + b \leq -c$ or $ax + b \geq c$	